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CORRELATIONS AMONG TWO-NUCLEON SCATTERING OBSERVABLES*

Yuriy Volkotrub, Jacek Golak, Roman Skibiński
Kacper Topolnicki, Henryk Witała

M. Smoluchowski Institute of Physics, Jagiellonian University
Łojasiewicza 11, 30-348 Kraków, Poland

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We use a covariance matrix of nucleon–nucleon (NN) potential parameters to investigate correlations among neutron–proton (np) scattering observables. To this end, we employ the up-to-date NN semilocal momentum-space regularized chiral interactions, solve the Lippman–Schwinger equation, and compute NN observables and their correlations. As a result, we present a systematic analysis of the correlation coefficients between the asymmetry P and the depolarisations A, A' observables at two incoming laboratory energies $E_{\rm lab} = 30$ MeV and $E_{\rm lab} = 65$ MeV.

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1. Introduction

Understanding of the nuclear force is the fundamental goal of nuclear physics. That force emerges from the strong interactions of quark and gluons described by QCD. Despite big progress made to establish the properties of nuclear forces, their detailed structure is not yet well known. This situation is a result of the highly non-perturbative behavior of QCD in the low-energy regime which significantly complicates the theoretical description of nuclear systems. Nowadays, there are numerous effective models of nuclear interactions applied in *ab initio* calculations which provide a description of nucleon–nucleon data set with χ^2/datum close to one, for example see Refs. [1–3]. A typical two-nucleon (2N) force model depends on free parameters, whose values are fixed from the NN data. For us, the most important example of such models is the new generation of the chiral interaction derived up to fifth-order (N^{4}LO+) of the chiral expansion using the semilocal regularization in momentum space (SMS) by the Bochum–Bonn group [1]. The

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choice of this model is dictated by the availability of the covariance matrix for the potential parameters. This allows us to systematically investigate correlations among NN scattering observables. A knowledge of these correlations could impact the procedures used to fix the free parameters of the NN force. While there are many experimental data (3691 neutron–proton data points up to 350 MeV [4] are used to fix NN potential parameters), most of them are the unpolarized cross sections or polarization observables with only one particle polarized in the initial state. Such a choice yields precise values of free parameters but models fixed in that way have only moderate predictive power in the 2N system. In addition, if correlated observables are used during the fixing procedure, the obtained values of parameters are biased. In this paper, we show two examples of the correlations of the NN scattering observables. We use the chiral interactions what allows us to investigate also a dependence of the correlation pattern on the order of chiral expansion.

We have been equipped by the authors of Ref. [1] with the mean values of the NN potential parameters and their correlation matrix for the chiral interaction at each order of chiral expansion. This allows us to sample 50 sets of potential parameters, also called low-energy constants (LECs), from the multivariate normal distribution and obtain many versions of the corresponding NN force. Next, the NN interaction V is inserted into the Lippmann–Schwinger equation in order to obtain the NN t-matrix operator

$$t = V + V\tilde{G}_0 t\,. \tag{1}$$

By solving Eq. (1), we get the NN on-shell scattering transition amplitude in the form of the M-matrix written in terms of the Wolfenstein parameters a, c, m, g, h and the Pauli matrices $\boldsymbol{\sigma}^1$ and $\boldsymbol{\sigma}^2$ for nucleons 1 and 2

$$\begin{aligned} M \;=\; & a + c\left(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}\right)\hat{\boldsymbol{N}} + m\left(\boldsymbol{\sigma}^{(1)}\hat{\boldsymbol{N}}\right)\left(\boldsymbol{\sigma}^{(2)}\hat{\boldsymbol{N}}\right) \\ & +(g+h)\left(\boldsymbol{\sigma}^{(1)}\hat{\boldsymbol{P}}\right)\left(\boldsymbol{\sigma}^{(2)}\hat{\boldsymbol{P}}\right) + (g-h)\left(\boldsymbol{\sigma}^{(1)}\hat{\boldsymbol{K}}\right)\left(\boldsymbol{\sigma}^{(2)}\hat{\boldsymbol{K}}\right)\,. \end{aligned} \tag{2}$$

The three unit momenta $\hat{K}, \hat{P}$ and $\hat{N}$ are defined through the initial, $\boldsymbol{q}$, and final, $\boldsymbol{q}'$, relative momenta of nucleons 1 and 2. From the M-matrix, all the NN scattering observables can be calculated. For more details, we refer to Ref. [5].

2. Results

For np scattering we computed, for each version of the potential, various NN scattering observables including depolarisations A, A' and asymmetry P [5] as well as their correlation coefficients at selected incoming nucleon energies and in the whole range of scattering angles. In Fig. 1, we present the angular dependence of the correlation coefficients for the above-mentioned NN observables as functions of the center-of-mass scattering angle $\theta_{\rm cm}$ at two laboratory kinetic energies $E_{\rm lab}$ = 30 MeV and $E_{\rm lab}$ = 65 MeV of the incident neutron. We employ the chiral SMS NN potentials at N^{2}LO, N^{3}LO, N^{4}LO and N^{4}LO+ with the cutoff parameter Λ = 450 MeV. As shown in Fig. 1, the asymmetry P is, in general, weakly correlated with both depolarisations, A and A' for the employed potentials and for both energies. The only cases of strong correlation appear at backward angles for the (P, A') pair at N^{2}LO at $E_{\rm lab}$ = 30 MeV and at $E_{\rm lab}$ = 65 MeV for all potentials. Additionally, we observe striking differences in the results for the correlation

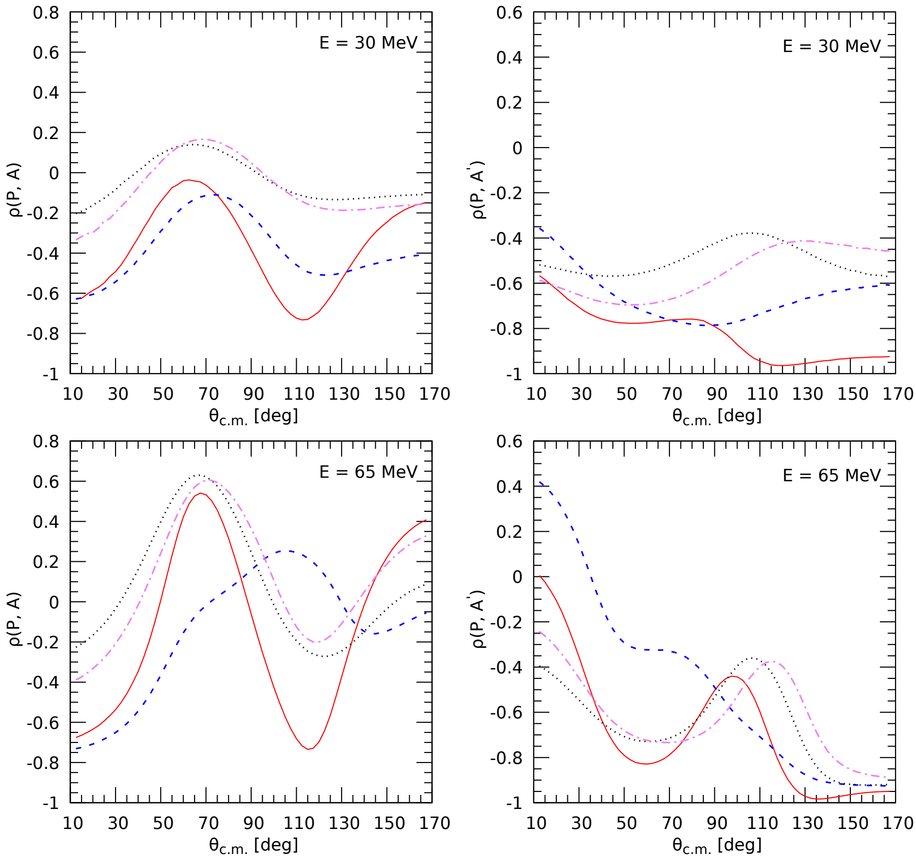

Fig. 1. (Color online) The angular dependence of correlations coefficients between selected np scattering observables [5]: P and A (top), P and A' (bottom), at the laboratory energy $E_{\rm lab}$ = 30 MeV (left) and 65 MeV (right). The solid (red), dashed (blue), dotted (orange), and dot-dashed (violet) lines represent predictions of the chiral SMS N^{2}LO, N^{3}LO, N^{4}LO and N^{4}LO+ forces, respectively.

coefficients obtained at different orders of chiral expansion. Only the correlation coefficients for the $(P,\ A)$ pair at N^4LO and N^4LO+ are close one to another. This reflects the similarity of predictions for these observables at N^4LO and N^4LO+. In the case of P and A', the predictions for correlation coefficients for N^4LO+ are slightly shifted with respect to ones at N^4LO. The observed behavior of correlation coefficients is presumably influenced by different numbers of LECs present at different orders of the chiral expansion. Explanation of the observed complex behavior of the correlation coefficients requires further investigations which we plan in the nearest future.

3. Summary

We showed that it is possible to analyze correlations among various NN observables using the covariance matrices provided with the models of the NN force from the Bochum–Bonn group. Consequently, we obtained the correlation coefficients for two selected pairs of the NN scattering observables and showed that the angular dependence of the correlation coefficients depends strongly on the order of chiral expansion as well as on the scattering energy. We hope that the results of this study will help to improve the effectiveness of the fitting procedure used to determine NN potential parameters.

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